

Classical BRST charges in reducible BRST-anti-BRST theories

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Abstract

We give a solution to the classical master equation of the Hamiltonian BRST-anti-BRST quantization scheme in the case of reducible gauge theories. Our approach does not require redefining constraints or reducibility functions. Classical BRST observables are also constructed.

1 Introduction

The BRST-anti-BRST symmetry in the Hamiltonian quantization of a gauge theory is generated by charges Ω^a , $a = 1, 2$, satisfying the master equation

$$\{\Omega^a, \Omega^b\} = 0. \quad (1)$$

For irreducible theories the general solution to (1) was given in [1] (see also [2]). Redefining constraints and reducibility functions in a theory with linearly dependent constraints one can bring them to a standard form. By using this fact the existence and uniqueness theorem for Ω^a in the case of reducible theories was proved [3],[4].

In the framework of the standard version of generalized canonical formalism, the BRST construction of gauge theories with linearly dependent generators was given in [5]. The global existence of the BRST charge was proved in [6]. Another construction of the BRST charge for a reducible

gauge theory was proposed in [7]. This construction yields the BRST charge without changing constraints or reducibility functions.

The goal of this paper is to present a solution to (1) in terms of the original constraints and reducibility functions in the case of gauge theories of any stage of reducibility. To discuss (1), it is convenient to treat $\Omega = (\Omega^a)$ as an element of a Poisson algebra of symmetric $\text{Sp}(2)$ tensors and to combine the Kozul-Tate differentials δ^a , $a = 1, 2$, into a single operator δ . Our approach is based on a special representation of δ . We find a coordinate system in the configuration space that brings δ to a linear derivation¹. This enables us to construct a generalized inverse of δ . Then (1) is solved by a simple iterative procedure. We also give a solution to the equation determining the classical BRST observables.

The paper is organized as follows. In section 2, we review the master equation, introduce the Poisson algebra of symmetric $\text{Sp}(2)$ tensors and rewrite (1) in terms of that algebra. In section 3, we introduce new variables in the configuration space and find a generalized inverse of δ . A solution to the master equation is given in section 4. In section 5, a realization of the observable algebra is described.

In what follows the Grassmann parity and new ghost number of a function X are denoted by $\epsilon(X)$ and $\text{ngh}(X)$, respectively. The Poisson superbracket in phase space $\Gamma = (P_A, Q^A)$, $\epsilon(P_A) = \epsilon(Q^A)$, is given by

$$\{X, Y\} = \frac{\partial X}{\partial Q^A} \frac{\partial Y}{\partial P_A} - (-1)^{\epsilon(X)\epsilon(Y)} \frac{\partial Y}{\partial Q^A} \frac{\partial X}{\partial P_A}.$$

Derivatives with respect to generalized momenta P_A are always understood as left-hand, and those with respect to generalized coordinates Q^A as right-hand ones. Indices of the symplectic group $\text{Sp}(2)$ are denoted by latin lowercase letters a_1, a_2, \dots . For a function $X_{a_1 a_2 \dots a_n}$

$$X_{\{a_1 a_2 \dots a_n\}} = X_{a_1 a_2 \dots a_n} + \text{cycl. perm. } (a_1, a_2, \dots, a_n).$$

2 Master equation for the BRST charge

Let $(p_i, q^i, i = 1, \dots, m)$ be the phase space coordinates, and let T_{α_0} , $\alpha_0 = 1, \dots, m_0$, $m_0 \leq 2m$, be the first class constraints which satisfy the following Poisson

¹ Any derivation that leaves a space of linear polynomials invariant is called a linear derivation.

brackets

$$\{T_{\alpha_0}, T_{\beta_0}\} = T_{\gamma_0} U_{\alpha_0 \beta_0}^{\gamma_0},$$

where $U_{\alpha_0 \beta_0}^{\gamma_0}$ are phase space functions. The constraints are assumed to be of definite Grassmann parity ϵ_{α_0} , $\epsilon(T_{\alpha_0}) = \epsilon_{\alpha_0}$.

We shall consider a reducible theory of L -th order. That is, there exist phase space functions

$$Z_{\alpha_{k+1}}^{\alpha_k}, \quad k = 0, \dots, L-1, \quad \alpha_k = 1, \dots, m_k,$$

such that at each stage the Z 's form a complete set,

$$Z_{\alpha_{k+1}}^{\alpha_k} \lambda^{\alpha_{k+1}} \approx 0 \Rightarrow \lambda^{\alpha_{k+1}} \approx Z_{\alpha_{k+2}}^{\alpha_{k+1}} \lambda^{\alpha_{k+2}}, \quad k = 0, \dots, L-2,$$

$$Z_{\alpha_L}^{\alpha_{L-1}} \lambda^{\alpha_L} \approx 0 \Rightarrow \lambda^{\alpha_L} \approx 0.$$

$$T_{\alpha_0} Z_{\alpha_1}^{\alpha_0} = 0, \quad Z_{\alpha_{k+1}}^{\alpha_k} Z_{\alpha_{k+2}}^{\alpha_{k+1}} = T_{\beta_0} A_{\alpha_{k+2}}^{\beta_0 \alpha_k}, \quad k = 1, \dots, L, \quad (2)$$

with

$$A_{\alpha_2}^{\alpha_0 \beta_0} = -(-1)^{\epsilon_{\alpha_0} \epsilon_{\beta_0}} A_{\alpha_2}^{\beta_0 \alpha_0}.$$

The weak equality \approx means equality on the constraint surface

$$\Sigma : \quad T_{\alpha_0} = 0.$$

An extended phase space of the theory under consideration is parametrized by the canonical variables

$$\Gamma = (P_A, Q^A) = (\xi_\alpha; P_{A'}, Q^{A'}), \quad (\xi_\alpha) = (p_i, q^i),$$

$$(P_{A'}, Q^{A'}) = (\mathcal{P}_{\alpha_s | a_1 \dots a_{s+1}}, c^{\alpha_s | a_1 \dots a_{s+1}}; \lambda_{\alpha_s | a_1 \dots a_s}, \pi^{\alpha_s | a_1 \dots a_s}; s = 0, \dots, L),$$

$$\mathcal{P}_{\alpha_s}, c^{\alpha_s} \in \mathcal{S}^{s+1}, \quad \lambda_{\alpha_s}, \pi^{\alpha_s} \in \mathcal{S}^s,$$

$$\lambda_{\alpha_s | a_1 \dots a_s} \Big|_{s=0} \equiv \lambda_{\alpha_0}, \quad \pi^{\alpha_s | a_1 \dots a_s} \Big|_{s=0} \equiv \pi^{\alpha_0}.$$

The Grassmann parities of the canonical variables are defined as follows:

$$\epsilon(\xi_\alpha) = \epsilon_\alpha, \quad \epsilon(\mathcal{P}_{\alpha_s|a_1\dots a_{s+1}}) = \epsilon(c^{\alpha_s|a_1\dots a_{s+1}}) = \epsilon_{\alpha_s} + s + 1,$$

$$\epsilon(\lambda_{\alpha_s|a_1\dots a_s}) = \epsilon(\pi^{\alpha_s|a_1\dots a_s}) = \epsilon_{\alpha_s} + s.$$

Variables of the extended phase space are assigned new ghost numbers by the rule

$$\text{ngh}(\xi_\alpha) = 0,$$

$$\text{ngh}(\mathcal{P}_{\alpha_s|a_1\dots a_{s+1}}) = \text{ngh}(c^{\alpha_s|a_1\dots a_{s+1}}) = s + 1,$$

$$\text{ngh}(\pi^{\alpha_s|a_1\dots a_s}) = -\text{ngh}(\lambda_{\alpha_s|a_1\dots a_s}) = s + 2.$$

Eq. (1) is supplied by the conditions

$$\epsilon(\Omega^a) = 1, \quad \text{ngh}(\Omega^a) = 1. \quad (3)$$

We shall seek Ω^a in the following form:

$$\Omega^a = \Omega_1^a + \Pi^a, \quad \Pi^a = \sum_{n \geq 2} \Omega_n^a, \quad \Omega_n^a \sim c^{n-m} \pi^m,$$

$$\begin{aligned} \Omega_1^a &= T_{\alpha_0} c^{\alpha_0|a} + \sum_{s=1}^L (\mathcal{P}_{\alpha_{s-1}|a_1\dots a_s} \delta_{s+1}^a Z_{\alpha_s}^{\alpha_{s-1}} + \mathcal{M}_{\alpha_s|a_1\dots a_{s+1}}^a) c^{\alpha_s|a_1\dots a_{s+1}} + \\ &+ \sum_{s=0}^L (\varepsilon^{ac} \mathcal{P}_{\alpha_s|ca_1\dots a_s} - [s/(s+1)] \lambda_{\alpha_{s-1}|a_1\dots a_{s-1}} \delta_s^a Z_{\alpha_s}^{\alpha_{s-1}} + \mathcal{N}_{\alpha_s|a_1\dots a_s}^a) \pi^{\alpha_s|a_1\dots a_s}, \quad (4) \end{aligned}$$

where $\mathcal{N}_{a_k}^a, \mathcal{M}_{a_k}^a$ are unknown functions of (\mathcal{P}, λ) , $\mathcal{N}_{a_0}^a = \mathcal{N}_{a_1}^a = \mathcal{M}_{a_1}^a = 0$. We assume that $\mathcal{N}_{a_k}^a$ and $\mathcal{M}_{a_k}^a$ only involves $\mathcal{P}_{\alpha_s}, \lambda_{\alpha_s}$ with $s \leq k - 2$. Eq. (3) implies

$$\mathcal{N}_{\alpha_k}^a|_{\mathcal{P}=\lambda=0} = 0, \quad \mathcal{M}_{\alpha_k}^a|_{\mathcal{P}=\lambda=0} = 0, \quad \Pi^a|_{\mathcal{P}=\lambda=0} = 0.$$

$\Omega = (\Omega^a)$ can be treated as an element of a Poisson algebra. Let \mathcal{S}^0 denote the space of smooth phase space functions, $\mathcal{S}^0 = C^\infty(\Gamma)$, and let \mathcal{S}^n , $n \geq 1$, be the space of the functions $X^{a_1 \dots a_n} \in C^\infty(\Gamma)$ that are symmetric under permutation of any indices. Given $X \in \mathcal{S}^q$ and $Y \in \mathcal{S}^p$, the symmetric product $X \circ Y$ is defined by

$$(X \circ Y)^{a_1 \dots a_n} = \frac{1}{n!} \sum_{\sigma \in \mathfrak{S}_n} X^{a_{\sigma(1)} \dots a_{\sigma(q)}} Y^{a_{\sigma(q+1)} \dots a_{\sigma(n)}}, \quad (5)$$

where $n = p + q$, and the summation is extended over the symmetric group of permutations of the numbers $1, \dots, n$. This product is supercommutative and associative [8]:

$$X \circ Y = (-1)^{\epsilon(X)\epsilon(Y)} Y \circ X, \quad X \circ (Y \circ Z) = (X \circ Y) \circ Z.$$

For any $X \in \mathcal{S}^q$, $Y \in \mathcal{S}^p$, we define the bracket $[\cdot, \cdot] : \mathcal{S}^q \times \mathcal{S}^p \rightarrow \mathcal{S}^{q+p}$ as

$$[X, Y] = \frac{\partial X}{\partial Q^A} \circ \frac{\partial Y}{\partial P_A} - (-1)^{\epsilon(X)\epsilon(Y)} \frac{\partial Y}{\partial Q^A} \circ \frac{\partial X}{\partial P_A}. \quad (6)$$

Let $\mathcal{S} = \bigoplus_{q=0}^{\infty} \mathcal{S}^q$. Products (5) and (6) transform \mathcal{S} into a graded Poisson algebra. One can directly verify that

$$[X, Y] = -(-1)^{\epsilon(X)\epsilon(Y)} [Y, X],$$

$$[X, Y \circ Z] = [X, Y] \circ Z + (-1)^{\epsilon(X)\epsilon(Y)} Y \circ [X, Z],$$

$$(-1)^{\epsilon(X)\epsilon(Z)} [X, [Y, Z]] + (-1)^{\epsilon(Y)\epsilon(X)} [Y, [Z, X]] + (-1)^{\epsilon(Z)\epsilon(Y)} [Z, [X, Y]] = 0,$$

$X, Y, Z \in \mathcal{S}$.

Let us define

$$\mathcal{V}^q = \{X \in \mathcal{S}^q : X|_{T=\mathcal{P}=\lambda=0} = 0\}$$

It is easily verified that $\mathcal{V} = \bigoplus_{q=1}^{\infty} \mathcal{V}^q$ is a Poisson subalgebra of \mathcal{S} , and $\Omega = (\Omega^a) \in \mathcal{V}^1$.

The bracket $\{\cdot, \cdot\}$ splits as

$$\{X, Y\} = \{X, Y\}_\xi + \{X, Y\}_\diamond - (-1)^{\epsilon(X)\epsilon(Y)} \{Y, X\}_\diamond,$$

where $\{.,.\}_\xi$ refers to the Poisson bracket in the original phase space and

$$\{X, Y\}_\diamond = \frac{\partial X}{\partial Q^{A'}} \frac{\partial Y}{\partial P_{A'}}.$$

Let $\delta^a : \mathcal{S}^0 \rightarrow \mathcal{S}^1$ be defined by

$$\delta^a = \{\Omega_1^a, .\}_\diamond. \quad (7)$$

Substituting (4) in (1), we get

$$\delta^a \Omega_1^b + \delta^b \Omega_1^a = 0, \quad (8)$$

$$\delta^{\{a} \Pi^{b\}} + F^{ab} + A^{\{a} \Pi^{b\}} + \{\Pi^a, \Pi^b\} = 0, \quad (9)$$

where $F^{ab} = \{\Omega_1^a, \Omega_1^b\}_{(p,q)}$, and the operator A^a is given by

$$A^a X = \{\Omega_1^a, X\}_\xi - (-1)^{\epsilon(X)} \{X, \Omega_1^a\}_\diamond, \quad X \in \mathcal{S}^0.$$

Eq. (8) can be written in the form

$$\delta^a \delta^b + \delta^b \delta^a = 0. \quad (10)$$

Given a pair of operators u^a , $a = 1, 2$, we define an operator $u : \mathcal{S}^n \rightarrow \mathcal{S}^{n+1}$ by

$$(uX)^a = u^a X, \quad n = 0,$$

$$(uX)^{a_1 \dots a_{n+1}} = \frac{1}{n+1} u^{\{a_1} X^{a_2 \dots a_{n+1}\}}, \quad n \geq 1. \quad (11)$$

Using (11) we rewrite (8) and (9) as

$$\delta \Omega_1 = 0, \quad (12)$$

$$\delta \Pi + F + A \Pi + \frac{1}{2} [\Pi, \Pi] = 0, \quad (13)$$

where $\Omega_1 = (\Omega_1^a)$, $\Pi = (\Pi^a)$, $F = (F^{ab})$. Eq. (12) expresses the nilpotency of δ :

$$\delta^2 = 0.$$

Let us define

$$W^a = \sum_{s=1}^L (\mathcal{M}_{\alpha_s|a_1\dots a_{s+1}}^a c^{\alpha_s|a_1\dots a_{s+1}} + \mathcal{N}_{\alpha_s|a_1\dots a_s}^a \pi^{\alpha_s|a_1\dots a_s}),$$

$$Q^{ab} = 2 \sum_{k=2}^L \left(\mathcal{P}_{\alpha_{k-2}|a_1\dots a_{k-1}} Z_{\alpha_{k-1}}^{\alpha_{k-2}} Z_{\alpha_k}^{\alpha_{k-1}} c^{\alpha_k|aba_1\dots a_{k-1}} + \right. \\ \left. + \frac{(k-1)}{k+1} \lambda_{\alpha_{k-2}|a_1\dots a_{k-2}} Z_{\alpha_{k-1}}^{\alpha_{k-2}} Z_{\alpha_k}^{\alpha_{k-1}} \pi^{\alpha_s|aba_1\dots a_{s-2}} \right),$$

and let

$$B^a X = \sum_{k=1}^L \left(\frac{\partial X}{\partial c^{\alpha_{k-1}|a_1\dots a_k}} Z_{\alpha_k}^{\alpha_{k-1}} c^{\alpha_k|aa_1\dots a_k} + \right. \\ \left. + (\varepsilon^{ab} \frac{\partial X}{\partial c^{\alpha_k|ba_1\dots a_k}} + \frac{k}{k+1} \frac{\partial X}{\partial \pi^{\alpha_{k-1}|a_1\dots a_{k-1}}} Z_{\alpha_k}^{\alpha_{k-1}} \delta_{a_k}^a) \pi^{\alpha_k|a_1\dots a_k} \right), \quad X \in \mathcal{S}^0.$$

Then (12) becomes

$$\delta W + Q + BW = 0. \quad (14)$$

For $L = 2$ (14) is satisfied by $\mathcal{M}_{\alpha_1}^a = \mathcal{N}_{\alpha_1}^a = 0$,

$$\mathcal{M}_{\alpha_2|a_1a_2a_3}^a = \frac{1}{6} (\mathcal{P}_{\alpha_0|a_1} \mathcal{P}_{\beta_0|a_2} \delta_{a_3}^a + \text{cycl.perm.}(a_1, a_2, a_3)) A_{\alpha_2}^{\beta_0\alpha_0} (-1)^{\epsilon_{\alpha_0}},$$

$$\mathcal{N}_{\alpha_2|a_1a_2}^a = \frac{1}{6} \lambda_{\alpha_0} \mathcal{P}_{\beta_0|\{a_1\}} \delta_{a_2}^a A_{\alpha_2}^{\beta_0\alpha_0} (-1)^{\epsilon_{\alpha_0}}. \quad (15)$$

We shall need two auxilliary equations. Let L denote the left-hand side of (12),

$$L = \delta\Omega_1 = \delta W + Q + BW.$$

By using the definition of δ , we get

$$\delta L = [\Omega_1, L]_{\diamond}. \quad (16)$$

where

$$[X, Y]_{\diamond} = \frac{\partial X}{\partial Q^{A'}} \circ \frac{\partial Y}{\partial P_{A'}}, \quad X, Y \in \mathcal{S}.$$

If (12) holds, then $\{\Omega^a, \Omega^b\} = R^{ab}$, where R^{ab} is the left-hand side of (13),

$$R^{ab} = \delta^{\{a} \Pi^{b\}} + F^{ab} + A^{\{a} \Pi^{b\}} + \{\Pi^a, \Pi^b\}.$$

From the Jacobi identity

$$\{\Omega^a, \{\Omega^b, \Omega^c\}\} + \text{cycl. perm. } (a, b, c) = 0$$

it follows that $[\Omega, R] = 0$, $\Omega = (\Omega^a)$, $R = (R^{ab})$, or equivalently

$$\delta R + AR + [\Pi, R] = 0. \quad (17)$$

Here we have used the relation

$$\{\Omega_1^a, .\} = \delta^a + A^a.$$

3 Generalized inversion of δ

For $k = L - 2$, (2) reads

$$Z_{\alpha'_{L-1}}^{\alpha_{L-2}} Z_{\alpha_L}^{\alpha'_{L-1}} + Z_{A_{L-1}}^{\alpha_{L-2}} Z_{\alpha_L}^{A_{L-1}} \approx 0, \quad (18)$$

where α'_{L-1}, A_{L-1} are index sets, such that $\alpha'_{L-1} \cup A_{L-1} = \alpha_{L-1}$, $|\alpha'_{L-1}| = |\alpha_L|$ and $\text{rank } Z_{\alpha_L}^{\alpha'_{L-1}} = |\alpha_L|$. For an index set $i = \{i_1, i_2, \dots, i_n\}$, we denote $|i| = n$. From (18) it follows that $\text{rank } Z_{\alpha_{L-1}}^{\alpha_{L-2}} = |\alpha_{L-1}| - |\alpha_L| = |A_{L-1}|$, and $\text{rank } Z_{A_{L-1}}^{\alpha_{L-2}} = |A_{L-1}|$.

One can split the index set α_{L-2} as $\alpha_{L-2} = \alpha'_{L-2} \cup A_{L-2}$, such that $|\alpha'_{L-2}| = |A_{L-1}|$, and $\text{rank } Z_{A_{L-1}}^{\alpha'_{L-2}} = |A_{L-1}|$. For $k = L - 3$, (2) implies

$$Z_{\alpha'_{L-2}}^{\alpha_{L-3}} Z_{A_{L-1}}^{\alpha'_{L-2}} + Z_{A_{L-2}}^{\alpha_{L-3}} Z_{A_{L-1}}^{A_{L-2}} \approx 0.$$

From this it follows that

$$\text{rank } Z_{A_{L-2}}^{\alpha_{L-3}} = \text{rank } Z_{\alpha_{L-2}}^{\alpha_{L-3}} = |\alpha_{L-2}| - |A_{L-1}| = |A_{L-2}|.$$

Using induction on k , we obtain a set of nonsingular matrices $Z_{A_k}^{\alpha'_{k-1}}$, $k = 2, \dots, L$, and a set of matrices $Z_{A_k}^{\alpha_{k-1}}$, $k = 1, \dots, L$, such that

$$\text{rank } Z_{A_k}^{\alpha_{k-1}} = Z_{\alpha_k}^{\alpha_{k-1}} = |A_k|.$$

Here $\alpha'_k \cup A_k = \alpha_k$, $k = 1, \dots, L-1$, and $A_L = \alpha_L$.

Eq. (2) implies

$$T_{\alpha'_0} Z_{A_1}^{\alpha'_0} + T_{A_0} Z_{A_1}^{A_0} = 0, \quad (19)$$

where $\alpha'_0 \cup A_0 = \alpha_0$, $|\alpha'_0| = |A_1|$, $\text{rank } Z_{A_1}^{\alpha'_0} = |A_1|$. From (19) it follows that T_{A_0} are independent. We assume that T_{A_0} satisfy the regularity conditions. It means that there are some functions $F_{\mathcal{A}}(\xi)$, $\mathcal{A} \cup A_0 = \alpha = (1, \dots, 2m)$, such that $(F_{\mathcal{A}}, G_{A_0})$ can be locally taken as new coordinates in the original phase space.

Let $f : A_{k+1} \rightarrow \alpha_k$, $k = 0, \dots, L-1$, be an embedding, $f(A_{k+1}) = A_{k+1} \in \alpha_k$, and let $\bar{\alpha}_k$ be defined by $\alpha_k = f(A_{k+1}) \cup \bar{\alpha}_k$. Since $|A_k| = |\bar{\alpha}_k|$, one can write $\bar{\alpha}_k = g(A_k)$ for some function g , and consequently

$$\alpha_k = f(A_{k+1}) \cup g(A_k), \quad k = 0, \dots, L-1.$$

Eq. (7) implies

$$\delta^a \xi_{\alpha} = 0, \quad \delta^a \mathcal{P}_{\alpha_0|b} = T_{\alpha_0} \delta_b^a, \quad \delta^a \lambda_{\alpha_0} = \varepsilon^{ab} \mathcal{P}_{\alpha_0|b},$$

$$\delta^a \mathcal{P}_{\alpha_s|a_1 \dots a_{s+1}} = \frac{1}{s+1} \mathcal{P}_{\alpha_{s-1}|\{a_1 \dots a_s\}} \delta_{a_{s+1}}^a Z_{\alpha_s}^{\alpha_{s-1}} + \mathcal{M}_{\alpha_s|a_1 \dots a_{s+1}}^a,$$

$$\delta^a \lambda_{\alpha_s|a_1 \dots a_s} = \varepsilon^{ab} \mathcal{P}_{\alpha_s|ba_1 \dots a_s} - \frac{1}{s+1} \lambda_{\alpha_{s-1}|\{a_1 \dots a_{s-1}\}} \delta_{a_s}^a Z_{\alpha_s}^{\alpha_{s-1}} + \mathcal{N}_{\alpha_s|a_1 \dots a_s}^a, \quad (20)$$

where $s = 1, \dots, L$.

We shall use the substitution (compare with (15))

$$\mathcal{M}_{\alpha_s|a_1 \dots a_{s+1}}^a = \frac{1}{s+1} \mathcal{M}_{\alpha_s|\{a_1 \dots a_s\}} \delta_{a_{s+1}}^a, \quad \mathcal{M}_{\alpha_s} \in \mathcal{S}^s,$$

$$\mathcal{N}_{\alpha_s|a_1 \dots a_s}^a = \frac{1}{s+1} \mathcal{N}_{\alpha_s|\{a_1 \dots a_{s-1}\}} \delta_{a_s}^a, \quad \mathcal{N}_{\alpha_s} \in \mathcal{S}^{s-1}. \quad (21)$$

From (21) it follows that

$$\mathcal{M}_{\alpha_s|a_1\dots a_s} = \frac{s+1}{s+2} \mathcal{M}_{\alpha_s|aa_1\dots a_s}^a, \quad \mathcal{N}_{\alpha_s|a_1\dots a_{s-1}} = \mathcal{N}_{\alpha_s|aa_1\dots a_{s-1}}^a.$$

Given $X_\alpha \in \mathcal{S}^{m+n}$, we denote

$$X_{\alpha|(m,n)} = X_{\alpha|\underbrace{1\dots 1}_m \underbrace{2\dots 2}_n}.$$

Then (20) becomes

$$\delta^a \xi_\alpha = 0, \quad \delta^1 \mathcal{P}_{\alpha_0|(r,t)} = rT_{\alpha_0} \quad \delta^2 \mathcal{P}_{\alpha_0|(r,t)} = tT_{\alpha_0},$$

$$\delta^1 \mathcal{P}_{\alpha_s|(r,t)} = \frac{r}{s+1} \left(\mathcal{P}_{\alpha_{s-1}|(r-1,t)} Z_{\alpha_s}^{\alpha_{s-1}} + \mathcal{M}_{\alpha_s|(r-1,t)} \right),$$

$$\delta^2 \mathcal{P}_{\alpha_s|(r,t)} = \frac{t}{s+1} \left(\mathcal{P}_{\alpha_{s-1}|(r,t-1)} Z_{\alpha_s}^{\alpha_{s-1}} + \mathcal{M}_{\alpha_s|(r,t-1)} \right),$$

$$\delta^1 \lambda_{\alpha_{s'}|(r',t')} = \mathcal{P}_{\alpha_{s'}|(r',t'+1)} - \frac{r'}{s'+1} \left(\lambda_{\alpha_{s'-1}|(r'-1,t')} Z_{\alpha_{s'}}^{\alpha_{s'-1}} - \mathcal{N}_{\alpha_{s'}|(r'-1,t')} \right),$$

$$\delta^2 \lambda_{\alpha_{s'}|(r',t')} = -\mathcal{P}_{\alpha_{s'}|(r'+1,t')} - \frac{t'}{s'+1} \left(\lambda_{\alpha_{s'-1}|(r',t'-1)} Z_{\alpha_{s'}}^{\alpha_{s'-1}} - \mathcal{N}_{\alpha_{s'}|(r',t'-1)} \right). \quad (22)$$

Here $s = 1, \dots, L$, $r+t = s+1$, $s' = 0, \dots, L$, $r'+t' = s'$.

Eq. (22) implies

$$(t+1)\delta^1 \mathcal{P}_{\alpha_{s+1}|(r+1,t)} = (r+1)\delta^2 \mathcal{P}_{\alpha_{s+1}|(r,t+1)},$$

$$(t'+1)(\delta^1 \lambda_{\alpha_{s'+1}|(r'+1,t')} - \mathcal{P}_{\alpha_{s'+1}|(r'+1,t'+1)}) =$$

$$= (r'+1)(\delta^2 \lambda_{\alpha_{s'+1}|(r',t'+1)} + \mathcal{P}_{\alpha_{s'+1}|(r'+1,t'+1)}).$$

Lemma. The derivations δ^a satisfying (10) are reducible to the form

$$\delta^a \xi'_\alpha = \delta^a \mathcal{P}'_{f(A_{s+1})|(r,t)} = 0,$$

$$\delta^1 \mathcal{P}'_{g(A_s)|(r,t)} = \frac{1}{t+1} \mathcal{P}'_{f(A_s)|(r-1,t)}, \quad \delta^2 \mathcal{P}'_{g(A_s)|(r,t)} = \frac{1}{r+1} \mathcal{P}'_{f(A_s)|(r,t-1)},$$

$$\delta^1 \lambda'_{f(A_{s'+1})|(r',t')} = -\frac{t'+1}{t'+2} \mathcal{P}'_{f(A_{s'+1})|(r',t'+1)},$$

$$\delta^2 \lambda'_{f(A_{s'+1})|(r',t')} = \frac{r'+1}{r'+2} \mathcal{P}'_{f(A_{s'+1})|(r'+1,t')},$$

$$\delta^1 \lambda'_{g(A_{s'})|(r',t')} = \frac{1}{t'+1} \lambda'_{f(A_{s'})|(r'-1,t')} + \mathcal{P}'_{g(A_{s'})|(r',t'+1)},$$

$$\delta^2 \lambda'_{g(A_{s'})|(r',t')} = \frac{1}{r'+1} \lambda'_{f(A_{s'})|(r',t'-1)} - \mathcal{P}'_{g(A_{s'})|(r'+1,t')}, \quad (23)$$

by the change of variables $(\xi, \mathcal{P}, \lambda) \rightarrow (\xi', \mathcal{P}', \lambda')$,

$$\xi'_{\mathcal{A}} = F_{\mathcal{A}}, \quad \xi'_{A_0} = T_{A_0},$$

$$\mathcal{P}'_{f(A_{s+1})|(r,t)} = (t+1) \delta^1 \mathcal{P}_{A_{s+1}|(r+1,t)}, \quad \mathcal{P}'_{g(A_s)|(r,t)} = \mathcal{P}_{A_s|(r,t)},$$

$$\lambda'_{f(A_{s'+1})|(r',t')} = (t'+1) (\delta^1 \lambda_{A_{s'+1}|(r'+1,t')} - \mathcal{P}_{A_{s'+1}|(r'+1,t'+1)}),$$

$$\lambda'_{g(A_{s'})|(r',t')} = \lambda_{A_{s'}|(r',t')}, \quad s, s' = 0, \dots, L-1, \quad g(A_L) = A_L. \quad (24)$$

To prove this statement we first observe that (24) is solvable with respect to the original variables. Assume that the functions $\xi_\alpha(\xi')$ have been constructed. Then from (24) it follows that

$$\mathcal{P}_{\alpha'_s|(r,t)} = \left(\frac{s+2}{(r+1)(t+1)} \mathcal{P}'_{f(A_{s+1})} - \mathcal{P}'_{g(A_s)} Z_{A_{s+1}}^{A_s} - \mathcal{M}'_{A_{s+1}} \right)_{|(r,t)} (Z^{(-1)})_{\alpha'_s}^{A_{s+1}}$$

$$\lambda_{\alpha'_{s'}|(r',t')} = - \left(\frac{s'+2}{(r'+1)(t'+1)} \lambda'_{f(A_{s'+1})} + \lambda'_{g(A_{s'})} Z_{A_{s'+1}}^{A_{s'}} - \mathcal{N}'_{A_{s'+1}} \right)_{|(r',t')} (Z^{(-1)})_{\alpha'_{s'}}^{A_{s'+1}}$$

$$\mathcal{P}_{A_s|(r,t)} = \mathcal{P}'_{g(A_s)|(r,t)}, \quad \lambda_{A_{s'}|(r',t')} = \lambda'_{g(A_{s'})|(r',t')}, \quad s, s' = 0, \dots, L.$$

Here and in what follows, for any $X(\xi, \mathcal{P}, \lambda, c, \pi)$ we denote by X' the function

$$X'(\xi', \mathcal{P}', \lambda', c, \pi) = X(\xi, \mathcal{P}, \lambda, c, \pi).$$

We have shown, therefore, that the variables $(\xi'_\alpha, \mathcal{P}'_{\alpha_s}, \lambda'_{\alpha_s}, s = 0, \dots, L,)$ are independent. Eq. (23) is a straightforward consequence of (22) and (24).

Let us define derivations σ_a , $a = 1, 2$, by

$$\sigma_a \xi'_{\alpha'} = 0, \quad \sigma_1 \xi'_A = \mathcal{P}'_{g(A_0)|(1,0)}, \quad \sigma_2 \xi'_A = \mathcal{P}'_{g(A_0)|(0,1)},$$

$$\sigma_1 \mathcal{P}'_{f(A_{s+1})|(r,t)} = \frac{(t+1)}{(s+2)} \left((r+1) \mathcal{P}'_{g(A_{s+1})|(r+1,t)} - \lambda'_{f(A_{s+1})|(r,t-1)} \right),$$

$$\sigma_2 \mathcal{P}'_{f(A_{s+1})|(r,t)} = \frac{(r+1)}{(s+2)} \left((t+1) \mathcal{P}'_{g(A_{s+1})|(r,t+1)} + \lambda'_{f(A_{s+1})|(r-1,t)} \right),$$

$$\sigma_1 \mathcal{P}'_{g(A_s)|(r,t)} = \frac{t}{s+1} \lambda'_{g(A_s)|(r,t-1)},$$

$$\sigma_2 \mathcal{P}'_{g(A_s)|(r,t)} = -\frac{r}{s+1} \lambda'_{g(A_s)|(r-1,t)},$$

$$\sigma_1 \lambda'_{f(A_{s'+1})|(r',t')} = \frac{(r'+1)(t'+1)}{(s'+2)} \lambda'_{g(A_{s'+1})|(r'+1,t')},$$

$$\sigma_2 \lambda'_{f(A_{s'+1})|(r',t')} = \frac{(r'+1)(t'+1)}{(s'+2)} \lambda'_{g(A_{s'+1})|(r',t'+1)},$$

$$\sigma_1 \lambda'_{g(A_{s'})|(r',t')} = \sigma_2 \lambda'_{g(A_{s'})|(r',t')} = 0.$$

Let N be a counting operator,

$$N \xi'_A = 0, \quad N \xi'_{A_0} = \xi'_{A_0}, \quad N \mathcal{P}'_{f(A_{s+1})} = \mathcal{P}'_{f(A_{s+1})}, \quad N \mathcal{P}'_{g(A_s)} = \mathcal{P}'_{g(A_s)},$$

$$N\mathcal{P}'_{g(A_s)} = \mathcal{P}'_{g(A_s)}, \quad N\lambda'_{g(A_{s'})} = \lambda'_{g(A_{s'})}, \quad N\lambda'_{f(A_{s'+1})} = \lambda'_{f(A_{s'+1})},$$

and let $M = \sigma_a \delta^a$. Then we have

$$\sigma_a \sigma_b + \sigma_b \sigma_a = 0, \quad \delta^a \sigma_b + \sigma_b \delta^a = N \delta_b^a,$$

$$N \delta^a = \delta^a N, \quad N \sigma_a = \sigma_a N,$$

$$M^2 \delta^a = N M \delta^a, \quad \sigma_a M^2 = N \sigma_a M,$$

$$M^n = (2^{n-1} - 1) N^{n-2} M^2 - (2^{n-1} - 2) N^{n-1} M, \quad n \geq 3. \quad (25)$$

With respect to the new coordinate system the condition $X \in \mathcal{V}$ becomes

$$X|_{\xi'_{A_0} = \mathcal{P}' = \lambda' = 0} = 0.$$

The space \mathcal{V} splits as

$$\mathcal{V} = \bigoplus_{k \geq 1} \mathcal{V}_k \quad (26)$$

with $NX = kX$ for $X \in \mathcal{V}_k$. Hence the operator N is invertible.

Let $\sigma : \mathcal{S}^n \rightarrow \mathcal{S}^{n-1}$ be defined by

$$\sigma X = 0, \quad n = 0, \quad (\sigma X)^{a_1 \dots a_n} = \sigma_a X^{a_1 \dots a_n a}, \quad n \geq 1.$$

One can directly verify that

$$\sigma^2 = 0, \quad \sigma M = (M - N)\sigma, \quad \delta M = (M + N)\delta,$$

$$(\sigma \delta + \delta \sigma)X = (nN + M)X, \quad X \in \mathcal{S}^n. \quad (27)$$

By using (25), we get

$$(nN + M)^{-1} = \frac{1}{n} N^{-1} - \frac{1}{n(n+2)(n+1)} ((n+3)MN^{-2} - M^2 N^{-3}), \quad n \geq 1.$$

Let $U : \mathcal{S}^n \rightarrow \mathcal{S}^n$ be defined by

$$U = \frac{1}{6}(11N^{-1} - 6MN^{-2} + M^2 N^{-3}), \quad n = 0,$$

$$U = (nN + M)^{-1}, \quad n \geq 1.$$

Then $\delta^+ = U\sigma$ is a generalized inverse of δ

$$\delta \delta^+ \delta = \delta. \quad (28)$$

From (27) it follows that

$$(\delta^+)^2 = 0,$$

and for any $X \in \mathcal{S}^n$, $n \geq 1$,

$$X = \delta^+ \delta X + \delta \Lambda \delta^+ X. \quad (29)$$

Here

$$\Lambda = \frac{1}{n(n+2)(n+1)} (n(n^2 + 4n + 6)I - (n-4)MN^{-1} - 2M^2N^{-2}),$$

and I is the identity map.

4 Solution of the master equation

Lowest order. Substitution $(\xi, \mathcal{P}, \lambda) \rightarrow (\xi', \mathcal{P}', \lambda')$ in (14) yields

$$\delta W' + Q' + B'W' = 0. \quad (30)$$

Applying $\delta\delta^+$ to (30) and using (28) we get

$$\delta W' + \delta\delta^+(B'W' + Q') = 0,$$

and consequently

$$W' + \delta^+(B'W' + Q') = Y', \quad (31)$$

where

$$Y' \in V^0, \quad \delta Y' = 0, \quad \epsilon(Y') = 1, \quad \text{ngh}(Y') = 1.$$

Solving (31), we get

$$W' = (I + \delta^+ B')^{(-1)}(Y' - \delta^+ Q'), \quad (32)$$

where

$$(I + \delta^+ B')^{(-1)} = \sum_{m \geq 0} (-1)^m (\delta^+ B')^m.$$

It remains to show that (32) satisfies (30). We shall use the approach of [9]. With respect to the new coordinate system (16) becomes

$$\delta L' = \{\Omega'^{(1)}, L'\}'_{\diamond}, \quad (33)$$

where

$$L' = \delta W' + B' W' + Q'.$$

If W' is a solution to (31), then

$$\delta^+ W' = \delta^+ Y',$$

and hence

$$\delta^+ L' = \delta^+ \delta W' + \delta^+ (W' + Q') = 0. \quad (34)$$

Consider (33) with condition (34). Applying δ^+ to (33), we get

$$L' = \delta^+ \{\Omega'^{(1)}, L'\}'_{\diamond},$$

from which by iterations it follows that $L' = 0$.

The functions $\mathcal{M}_{a_s}^a, \mathcal{N}_{a_s}^a, s = 1, \dots, L$, are found by substituting (24) in

$$\mathcal{M}_{a_s}^a(\xi, \mathcal{P}_{a_0}, \dots, \mathcal{P}_{a_{s-2}}, \lambda_{\alpha_0}, \dots, \lambda_{\alpha_{s-2}}) = \mathcal{M}_{\alpha_s}^{'a}(\xi', \mathcal{P}'_{a_0}, \dots, \mathcal{P}'_{\alpha_{s-2}}, \lambda'_{\alpha_0}, \dots, \lambda'_{\alpha_{s-2}}),$$

$$\mathcal{N}_{a_s}^a(\xi, \mathcal{P}_{a_0}, \dots, \mathcal{P}_{a_{s-2}}, \lambda_{\alpha_0}, \dots, \lambda_{\alpha_{s-2}}) = \mathcal{N}_{a_s}^{'a}(\xi', \mathcal{P}'_{a_0}, \dots, \mathcal{P}'_{a_{s-2}}, \lambda'_{\alpha_0}, \dots, \lambda'_{\alpha_{s-2}}),$$

where

$$\mathcal{M}_{a_s| (r, s+1-r)}^{'a} = \frac{r!(s+1-r)!}{(s+1)!} \frac{\partial W^{'a}}{\partial \mathcal{C}^{a_s| (r, s+1-r)}},$$

$$\mathcal{N}_{a_s| (r, s-r)}^{'a} = \frac{r!(s-r)!}{s!} \frac{\partial W^{'a}}{\partial \pi^{a_s| (r, s-r)}}.$$

Here we have used the relation

$$W'^a = \sum_{s=1}^L \left(\sum_{r=0}^{s+1} \frac{(s+1)!}{r!(s+1-r)!} \mathcal{M}'_{\alpha_s|(r,s+1-r)} c^{\alpha_s|(r,s+1-r)} + \right. \\ \left. + \sum_{r=0}^s \frac{s!}{r!(s-r)!} \mathcal{N}'_{\alpha_s|(r,s-r)} \pi^{\alpha_s|(r,s-r)} \right).$$

Assume that $\mathcal{M}_{a_s}^a, \mathcal{N}_{a_s}^a, s \leq k$, have been constructed. It follows from (22) and (24) that the variables $(\xi', \mathcal{P}'_{a_0}, \dots, \mathcal{P}'_{a_{k-1}}, \lambda'_{\alpha_0}, \dots, \lambda'_{\alpha_{k-1}})$ depend only on the functions $\mathcal{M}_{a_s}^a, \mathcal{N}_{a_s}^a$ with $s \leq k$, and therefore $\mathcal{M}_{a_{k+1}}^a, \mathcal{N}_{a_{k+1}}^a$ are easily computed. The functions $\mathcal{N}_{a_{k+1}}^a, \mathcal{M}_{a_{k+1}}^a$ only involves $\mathcal{P}_{\alpha_s}, \lambda_{\alpha_s}, s \leq k-1$, in agreement with the above assumption.

Higher orders. In the coordinate system $(\xi', \mathcal{P}', c, \lambda', \pi)$ (13) becomes

$$\delta \Pi' + F' + A \Pi' + \frac{1}{2} [\Pi', \Pi']' = 0. \quad (35)$$

Applying the operator $\delta \delta^+$ to (35), we get

$$\delta \Pi' + \delta \delta^+ (F' + A \Pi' + \frac{1}{2} [\Pi', \Pi']') = 0. \quad (36)$$

From (36) it follows that

$$\Pi' = \Upsilon - \delta^+ (F' + A \Pi' + \frac{1}{2} [\Pi', \Pi']'), \quad (37)$$

where

$$\Upsilon \in V^1, \quad \delta \Upsilon = 0, \quad \Upsilon = \sum_{n \geq 2} \Upsilon^{(n)}, \quad \Upsilon^{(n)} \sim c^{n-m} \pi^m.$$

If Π' is a solution to (37) then

$$\delta^+ \Pi' = \delta^+ \Upsilon. \quad (38)$$

Let us show that a solution to (37) satisfies (35). Changing variables in (17) from $(\xi, \mathcal{P}, \lambda)$ to $(\xi', \mathcal{P}', \lambda')$, we get

$$\delta R' + A R' + [\Pi', R']' = 0. \quad (39)$$

Consider (39), where Π' is a solution to (37), with the boundary condition

$$\delta^+ R' = 0. \quad (40)$$

Applying δ^+ to (39), we get

$$R' + \delta^+(AR' + [\Pi', R']') = 0.$$

From this by iterations it follows that $R' = 0$. For checking (40) we have

$$\delta^+ R' = \delta^+ \delta \Pi' + \delta^+(F' + A\Pi' + \frac{1}{2}[\Pi', \Pi']') = \delta^+ \delta \Pi' + \Upsilon - \Pi',$$

and therefore by (29) and (38), $\delta^+ R = 0$.

Let $\langle ., . \rangle : S^1 \times S^1 \rightarrow S^1$ be defined by

$$\langle X_1, X_2 \rangle = -\frac{1}{2}(I + \delta^+ A)^{-1} \delta^+ ([X_1, X_2] + [X_2, X_1]), \quad (41)$$

where

$$(I + \delta^+ A)^{-1} = \sum_{m \geq 0} (-1)^m (\delta^+ A)^m.$$

Then we have

$$\Pi' = \Pi_0 + \frac{1}{2} \langle \Pi', \Pi' \rangle, \quad (42)$$

where

$$\Pi_0 = (I + \delta^+ A)^{-1} (\Upsilon - \delta^+ F').$$

Iterating (42) we can construct Π' and, consequently, the BRST charge Ω . The first two terms are

$$\Pi' = \Pi_0 + \frac{1}{2} \langle \Pi_0, \Pi_0 \rangle + \dots$$

5 Observables

Let P denote the Poisson algebra of first class functions,

$$P = \{f(\xi) \mid \{f, T_\alpha\} \approx 0\},$$

and let

$$J = \{u(\xi) \mid u \approx 0\}.$$

Elements of the Poisson algebra P/J are called classical observables.

BRST observables are determined by solutions to the equation

$$[\Omega, \Phi] = 0, \quad \Phi \in S^0. \quad (43)$$

With respect to the variables $(\xi', \mathcal{P}', c, \lambda', \pi)$ (43) becomes

$$[\Omega', \Phi']' = 0. \quad (44)$$

The boundary conditions read

$$\text{ngh}(\Phi') = 0, \quad \Phi'|_{\mathcal{P}'=c=\lambda'=\pi=0} = \Phi_0, \quad \bar{\sigma}(\Phi' - \Phi_0) = 0, \quad (45)$$

where $\Phi_0(\xi') \in P$, $\bar{\sigma} = \epsilon^{ab}\sigma_a\sigma_b$.

The function Φ' can be written as

$$\Phi' = \Phi_0 + K, \quad K = \sum_{n \geq 1} \Phi^{(n)}, \quad \Phi^{(n)} \sim c^{n-m} \pi^m. \quad (46)$$

Substituting (46) in (44), we get

$$\delta K + [\Omega', \Phi_0]' + AK + [\Pi', K]' = 0 \quad (47)$$

or equivalently,

$$K + \delta^+([\Omega, \Phi_0]' + AK + [\Pi', K]') = \mathcal{Z}, \quad (48)$$

where

$$\mathcal{Z} \in S^0, \quad \delta \mathcal{Z} = 0, \quad \text{ngh}(\mathcal{Z}) = 0.$$

Let us denote $\bar{\delta} = \epsilon_{ab}\delta^a\delta^b$. Then

$$\bar{\delta}\bar{\sigma} - \bar{\sigma}\bar{\delta} = 4N^2 - 2MN,$$

from which it follows that for any $X \in S^0$

$$X = \frac{1}{2}MN^{-1}X + \frac{1}{4}(\bar{\delta}\bar{\sigma} - \bar{\sigma}\bar{\delta})N^{-2}X. \quad (49)$$

The boundary conditions (45) imply $\bar{\sigma}K = 0$, and therefore $\bar{\sigma}\mathcal{Z} = 0$, since $\bar{\sigma}\delta^+ = 0$. By using (49) we get $\mathcal{Z} = 0$. Solving (48) for K yields

$$K = -(I + \delta^+(A + \text{ad } \Pi))^{(-1)}\delta^+[\Omega, \Phi_0]. \quad (50)$$

We must now show that (48) satisfies (47).

The Jacobi identities for the functions Ω'^a, Φ' read

$$\{\Omega'^a, \{\Omega'^b, \Phi'\}'\}' + \{\Omega'^b, \{\Omega'^a, \Phi'\}'\}' = 0. \quad (51)$$

Let $G = (G^a)$ denote left-hand side of (47),

$$G = \delta K + [\Omega', \Phi_0]' + AK + [\Pi', K]'. \quad (52)$$

Then (51) becomes

$$\delta G + AG + [\Pi', G]' = 0. \quad (53)$$

It is easily verified that if K satisfies (48) then $\delta^+K = \delta^+\Upsilon$, and

$$\delta^+G = 0. \quad (54)$$

Consider equation (52) and boundary condition (53). By using (49), we get

$$G = -\delta^+(AG + [\Pi', G]').$$

From this it follows that $G = 0$.

We conclude that the solution to (44), (45) is given by

$$\Phi' = \mathcal{L}\Phi_0, \quad (55)$$

where

$$\mathcal{L} = I - (I + \delta^+(A + \text{ad } \Pi))^{(-1)}\delta^+[\Omega', \cdot]'. \quad (56)$$

The operator \mathcal{L} is invertible. The inverse \mathcal{L}^{-1} is given by

$$\mathcal{L}^{-1}\Phi' = \Phi'|_{\mathcal{P}'=c=\lambda'=\pi=0}.$$

Eq. (54) establishes a one-to-one correspondence between first class functions and solutions to (44), (45).

Let $\mathcal{L}(D)$ denote the image of $D \subset P$ under the mapping \mathcal{L} . For $\Phi'_1, \Phi'_2 \in \mathcal{L}(P)$

$$\{\Phi'_1, \Phi'_2\}'|_{\mathcal{P}'=c=\lambda'=\pi=0} = \{\Phi'_1|_{\mathcal{P}'=c=\lambda'=\pi=0}, \Phi'_2|_{\mathcal{P}'=c=\lambda'=\pi=0}\}',$$

$$(\Phi'_1\Phi'_2)|_{\mathcal{P}'=c=\lambda'=\pi=0} = \Phi'_1|_{\mathcal{P}'=c=\lambda'=\pi=0} \Phi'_2|_{\mathcal{P}'=c=\lambda'=\pi=0}.$$

This means that $\mathcal{L}(P)$ and P are isomorphic as Poisson algebras, and therefore $\mathcal{L}(P)/\mathcal{L}(J)$ gives a realization of classical observables.

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